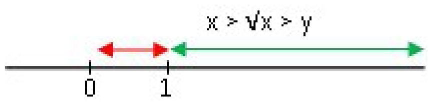
This week, let's look at some more properties of exponents and roots. Using a high level data sufficiency question, we will see how a number x is related to √x and to x^3.

Question: Is x > y?

Statement 1: √x > y
Statement 2: x^3 > y

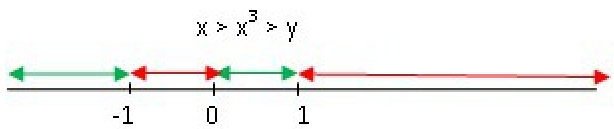
It is one of those gorgeous questions that seem very simple at first but surprise you later.

The question asks us whether x is greater than y, but the statements tell us the relation between √x, x^3 and y. If we know that √x is greater than y, when can we say that x is certainly greater than y too? If we know that x is greater than √x, then we can say for sure that x is greater than y too. Is x always greater than √x? No. Look at the diagram given below.



√x is not defined for negative values of x so let's ignore the section to the left of 0. When the value of x lies between 0 and 1, x is less than √x (for example: when √x = 1/2, x = 1/4). When the value of x is greater than 1, x is greater than √x (for example: when √x = 2, x = 4).

Similarly, let's look at the relation between x^3 and x. If we know that x^3 is greater than y, when can we say that x is certainly greater than y too? If we know that x is greater than x^3, then we can say for sure that x is greater than y too. Is x always greater than x^3? No. Look at the graph below.



When the value of x lies between -1 and 0 or in the region greater than 1, x is less than x^3 (for example: when x = 2, x^3 = 8). When the value of x lies in the region less than -1 or between 0 and 1, x is greater than x^3 (for example: when x = 1/2, x^3 = 1/8)

Let's look at the statements now

Statement 1: √x > y

Since √x is not defined for negative x, we get that x >= 0. As we saw in the first graph above, for some values, x is greater than √x, for others, x is less than √x. When x is less than √x, x may not be greater than y. So this statement alone is not sufficient.

Statement 2: x^3 > y

As we saw in the second graph above, for some values, x is greater than x^3, for others, x is less than x^3. When x is less than x^3, x may not be greater than y. So this statement alone is not sufficient.

Using both the statements together, we know that x >= 0.

When x lies between 0 and 1, we know that $x >= x^3$. Since statement (2) says that $x^3 > y$, we can say that $x > y$. When x is greater than 1, we know that $x > \sqrt{x}$. Since statement (1) says that $\sqrt{x} > y$, we can deduce that $x > y$. Therefore, for all possible values of x, we can say that $x > y$. Together the statements are sufficient. Answer (C).

It is important to understand these relations. This concept is very useful, especially for GMAT Algebra!